THE LAWS OF MOTION UNDER CONSTANT POWER.

LLOYD W. TAYLOR,
Oberlin College.

The laws of motion under constant *force* form the door of introduction to the subject of Dynamics, for most elementary Physics students, and properly so. In these days of steam, electric and gasoline transportation, the laws of motion under constant *power* would seem to be only second in importance. This opinion is evidently not shared by authors of textbooks on mechanics, for in the course of several months search the writer has been unable to find a single reference to such a subject. In the meantime he has investigated the laws of this type of motion, and found that they were both surprisingly simple and remarkably easy of deduction.

The laws of motion under constant *force* may be completely described by three equations:

$$v = at \qquad s = \tfrac{1}{2}at^2 \qquad v = \sqrt{2as}$$

These equations are built on the assumption that the initial values of the variables are zero. The acceleration a is of course a constant, the value of which may be found from Newton's second law of motion, $f = ma$.

In the laws of motion under constant *power*, four variables (s, v, a and t) are involved, instead of three as in the preceding case. Six equations, each involving two of the variables, are necessary to completely describe the laws of motion. This is a consequence of the fact that the number of possible combinations of four things taken two at a time is six.

These six equations may be conveniently stated as follows:

$$a = \frac{P}{mv} \quad (1) \qquad v = \sqrt{\frac{2Pt}{m}} \quad (2) \qquad s = \sqrt{\frac{8Pt^3}{9m}} \quad (3)$$

$$a = \sqrt{\frac{P}{2mt}} \quad (4) \qquad s = \frac{mv^3}{3P} \quad (5) \qquad a = \sqrt[3]{\frac{P^2}{3m^2s}} \quad (6)$$

As before, the initial values of the variables are assumed to be zero. This is simply equivalent to saying that both distance

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and time are respectively measured from the point and instant of starting, which would, of course, be the natural thing to do. *P*, representing the constant power applied, must, like *f* in the preceding case, be measured in absolute units.

A deduction of these laws follows: By definition, $P=fv$ and $f=ma$. Combining to eliminate *f*, $a=\frac{P}{mv}$ (1.)

Substitute $a=\frac{dv}{dt}$ and restate equation 1 as follows: $vdv=\frac{P}{m}dt$ whence, integrating between the limits 0 to *v* and 0 to *t* respectively, $v=\sqrt{\frac{2\,Pt}{m}}$ (2.)

Substitute $v=\frac{ds}{dt}$ and restate equation 2 as follows: $ds=\sqrt{\frac{2\,Pt}{m}}\,dt$, whence, integrating between proper limits

$$s=\sqrt{\frac{8\,Pt^3}{9\,m}} \quad (3.)$$

Substitute equation 2 into equation 1 to eliminate *v*, whence,

$$a=\sqrt{\frac{P}{2mt}} \quad (4.)$$

Substitute equation 2 into equation 3 to eliminate *t*, whence,

$$s=\frac{mv^3}{3P} \quad (5.)$$

Substitute equation 5 into equation 1 to eliminate *v*, whence,

$$a=\sqrt[3]{\frac{P^2}{3m^2s}} \quad (6.)$$

In case it is not desired to subject the conclusions to the limitations involved in the assumption of zero initial values, the corresponding relations are readily deduced by the simple expedient of changing the values of the limits in the two integrations above.

Three of these relations are deduced without the use of calculus. Equation 1 was so derived above. In addition, one may utilize the principle of Conservation of Energy by writing

$$Pt=\tfrac{1}{2}\,m\,v^2 \quad \text{whence} \quad v=\sqrt{\frac{2\,Pt}{m}} \quad (2.)$$

Substitution of equation 2 into equation 1 gives equation 4, as before. The other three equations do not appear to readily yield to the freshman type of attack. It is a simple matter to show, however, by elementary methods, that the value of the distance in terms of the velocity, (equation 5) must lie between $\frac{m\,v^3}{2\,P}$ and $\frac{m\,v^3}{4\,P}$. And if it be considered allowable to conclude that the true expression is $\frac{m\,v^3}{3\,P}$ then the remaining equations follow.

A brief glance at some numerical cases may not be entirely without interest. Calculation shows that a fifty horsepower automobile weighing two tons will travel 444 feet the first ten seconds from a standing start, will acquire a velocity of 45.4 miles per hour and will possess at the conclusion of the ten seconds an acceleration of 2.3 miles per hour per second. For five seconds the corresponding figures are 156.6, 33.1, and 3.3, and for one second, 13.9, 14.9 and 7.35.

Perhaps it is unnecessary to add the caution that in these equations power must be expressed in absolute units. In the metric system this would be ergs per second, and in the English system foot poundals per second. If the British Engineers system is used, mass will of course be expressed in "slugs" and power in foot pounds per second.

This problem assumes an entirely different aspect if, in an attempt to represent the facts more closely, we introduce terms to account for friction, air resistance and the pull of gravity in going up or down grade. It then becomes essentially a problem of motion in a resisting medium, friction and gravity representing constant forces and air resistance a force closely proportional to the square of the velocity. Introduction of these factors leads to a differential equation which anyone may be pardoned for not easily solving. A discussion of it may properly form the basis for a later report. In the meantime, we have before us the laws of motion for constant *power*, corresponding to the three classical laws for constant *force*, which have always been so much in evidence in our work in mechanics.